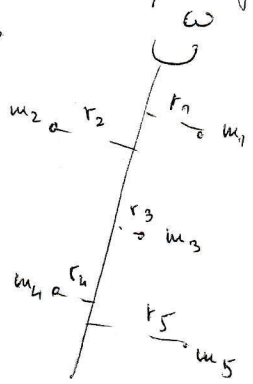
Energia kinetyczna ruchu obrotowego

"Bryła sztywna"

$$E_k = \frac{m v^2}{2} \qquad v = \omega r \quad \Rightarrow \quad E_k = \frac{m \omega^2 r^2}{2}$$

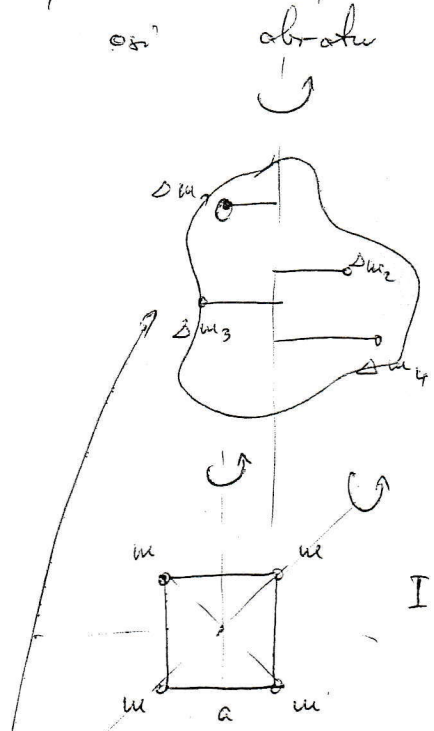
Jeżeli ciało jest sztywne tj. punkt bryły zmieniają odl. od osi obrotu $\Rightarrow$



$$E_k = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots$$
$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots =$$
$$= \frac{1}{2} \left(m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2 \right) \omega^2 =$$
$$= \frac{1}{2} I \omega^2$$

I – moment bezwładności

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$



$I = ?$ (tensor)

$$\hat{I} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

$$\boxed{\vec{L} = \hat{I} \vec{\omega}}$$

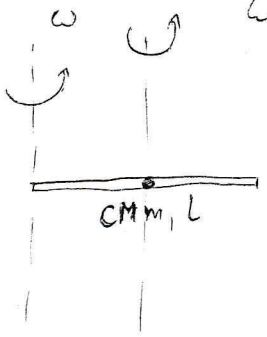
$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

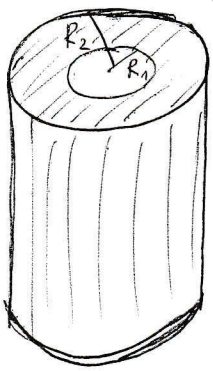
$$\boxed{I = \int r^2 \, dm} \qquad \text{tj.} \quad \sum \longrightarrow \int$$

$$dm = \rho \, dV$$

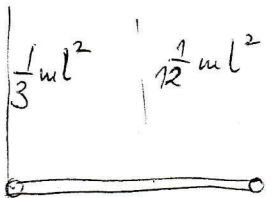
tj. $m = \rho V$

$\rho = \text{const}$ (jednorodne ciało)





$$I = \int r^2 \, dm$$



$$I_{xy} = I_{yx} = -\sum_k m_k x_k y_k$$
$$I_{xz} = I_{zx} = -\sum_k m_k x_k z_k$$
$$I_{yz} = I_{zy} = -\sum_k m_k y_k z_k$$

obręcz	$m R^2$
walec	$\frac{1}{2} m R^2$
kula	$\frac{2}{5} m R^2$
powłoka sfer.	$\frac{2}{3} m R^2$
prostopadłościan	$\frac{1}{2} m (a^2 + b^2)$

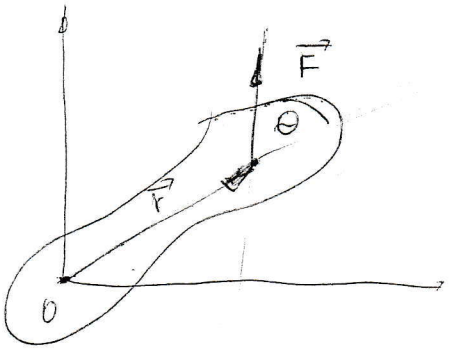
$$I_{ij} = \sum_k m_k \left(r_k^2 \delta_{ij} - r_{ki} r_{kj} \right)$$

$r_1 = x \; ; \; r_2 = y \; ; \; r_3 = z$

$$I_{xx} = \sum_k m_k \left(y_k^2 + z_k^2 \right) = \sum_k m_k \left(r_k^2 - x_k^2 \right)$$
$$I_{yy} = \sum_k m_k \left(x_k^2 + z_k^2 \right) = \sum_k m_k \left(r_k^2 - y_k^2 \right)$$
$$I_{zz} = \sum_k m_k \left(y_k^2 + z_k^2 \right) = \sum_k m_k \left(r_k^2 - z_k^2 \right)$$

$$\boxed{I_{ij} = \int_V \rho(x,y,z) \left(r^2 \delta_{ij} - r_i r_j \right) dV}$$

Dynamika ciała sztywnego



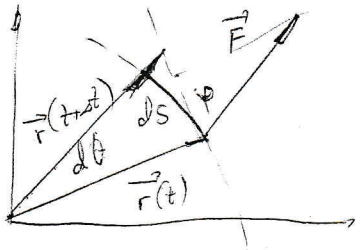
$\vec{M} = \vec{r} \times \vec{F}$ $\quad |\vec{M}| = M = r F \sin\theta$

$M = r F_\perp$ lub $M = F r_\perp$

Praca przy obracaniu

$d\vec{s} = r\, d\theta$

$dW = \vec{F} \cdot d\vec{s} = F \cos\varphi \; r \, d\theta =$



$(F\cos\varphi)\, r = M$ (moment siły)

$dW = M\, d\theta$

$W = \int M\, d\theta$

moc w ruchu obrotowym

w ruchu postępowym

$P = \frac{dW}{dt} = M \frac{d\theta}{dt} = M\omega$

↖ jeśli $M = \text{const}$

$dW = F(x)\, dx$

$\frac{dW}{dt} = F \frac{dx}{dt} = F v$

Ruch postępowy	Ruch obrotowy wokół osi nieruch.
x	θ
$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\varepsilon = \frac{d\omega}{dt}$
~~M~~ m	I
$F = ma$	$M = I\varepsilon$
$W = \int F\, dx$	$W = \int M\, d\theta$
$\frac{1}{2} m v^2$	$\frac{1}{2} I \omega^2$
$P = Fv$	$P = M\omega$
~~Mv~~ mv	$I\omega$

$dW = M\,d\theta$

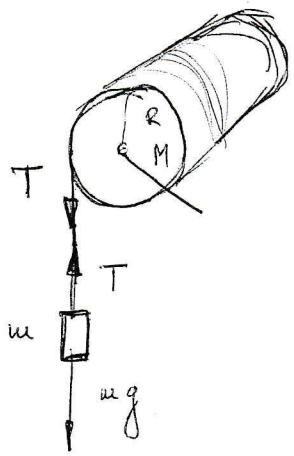
$\frac{dW}{dt} = M\frac{d\theta}{dt} = M\omega$

$dW = dE_k \Rightarrow \frac{dW}{dt} = \frac{dE_k}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right)$

$\frac{dW}{dt} = \frac{1}{2}I\,\omega\,2\frac{d\omega}{dt} = I\omega\frac{d\omega}{dt} = I\omega\varepsilon$

$M\omega = I\omega\varepsilon \qquad \vec{M} \parallel \vec{\varepsilon}$

$$\boxed{M = I\varepsilon}$$



dane:
$M = 2.5\,kg \qquad R = 20\,cm$

$mg - T = ma$

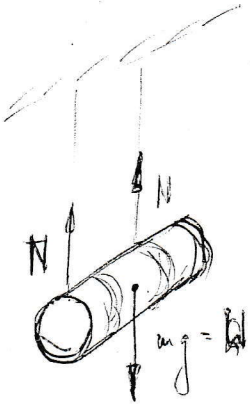
$TR = I\varepsilon = \frac{1}{2}MR^2\varepsilon \Rightarrow \varepsilon = \frac{2TR}{mR^2}$

$a = R\varepsilon = \frac{2T}{m}$

$T = \frac{Ia}{R^2} \qquad mg = a\left(m + \frac{I}{R^2}\right) = a\left(m + \frac{\frac{1}{2}MR^2}{R^2}\right)$

$a = \frac{mg}{m + \frac{1}{2}M} = \frac{2m}{(2m+M)}g$

$T = \frac{M\,2m}{2(2m+M)}g = \frac{mMg}{(2m+M)}$



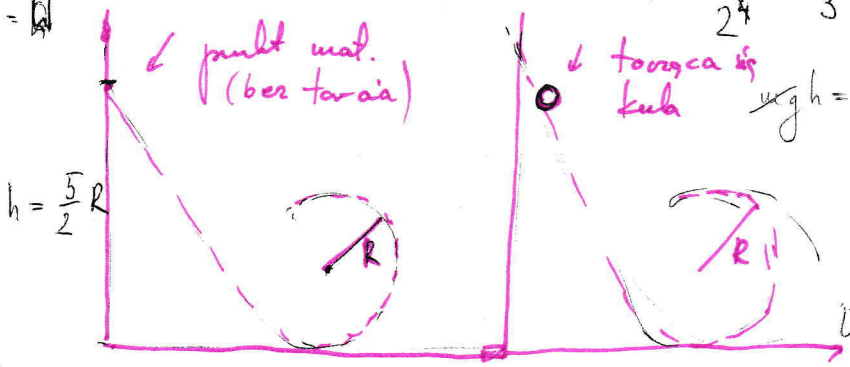
$W - 2N = ma \Rightarrow W = ma + \frac{1}{2}ma = \frac{3}{2}ma$

$2NR = I\frac{a}{R}$

$N = \frac{1}{4}mR^2\frac{a}{R^2} = \frac{1}{4}ma$

$\{a = \frac{2}{3}g\}$

$N = \frac{1}{4}m\frac{2}{3}g = \frac{W}{6}$

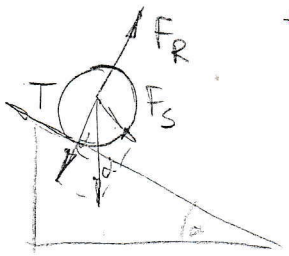


$mgh = \frac{mv^2}{2} + \frac{\frac{2}{5}mR^2v^2}{2R^2} = \frac{7}{10}v^2$

$h = 2.7R$

$mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$

$v = \omega r \qquad I = \frac{2}{5}mR^2$



$\begin{cases} mg\sin\alpha - T = ma \\ TR = \varepsilon I = \frac{a}{R}I \end{cases}$ bo $\vec{a} = \vec{\varepsilon}\times\vec{r} \qquad \vec{\varepsilon} \perp \vec{r}$

$T = \frac{a}{R^2}I = \frac{a}{R^2}\frac{1}{2}mR^2$

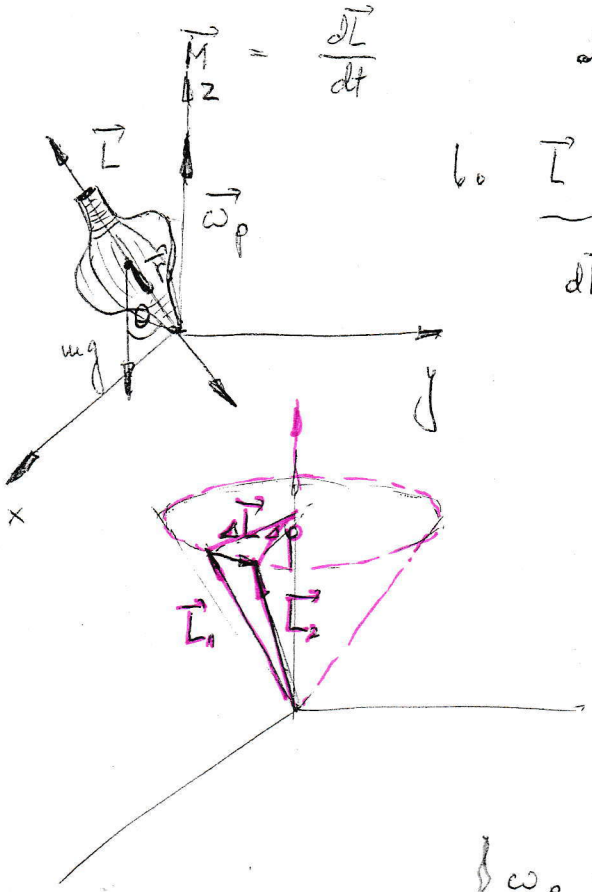
$a = \varepsilon R$

$I = \frac{1}{2}mR^2$

$mg\sin\alpha - \frac{1}{2}ma = ma$

$$\boxed{a = \frac{2}{3}g\sin\alpha}$$

Zasada zachowania momentu pędu oraz **Precesja** żyroskopu



$\vec{M} = \frac{d\vec{L}}{dt}$ ale $\vec{M} = I\vec{\varepsilon}$

bo $\vec{L} = I\vec{\omega}$

$d\vec{L} \parallel \vec{M}$

$\vec{M} = \vec{r} \times m\vec{g} = \vec{r} \times \vec{Q}$

$\Delta\vec{L} = \vec{M}\Delta t$

$\omega_p = \frac{\Delta\varphi}{\Delta t}$

$\Delta\varphi = \frac{\Delta L}{L\sin\theta} = \frac{M\Delta t}{L\sin\theta}$

$\omega_p = \frac{M\Delta t}{L\sin\theta\,\Delta t} =$

$M = r\,mg\sin(\pi - \theta) = r\,mg\sin\theta$

$$\omega_p = \frac{r\,mg\sin\theta}{L\sin\theta} = \frac{mgr}{L} = \frac{mgR}{I_s\omega}$$

Jeśli $\vec{M}_{zew.} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = const$

$\vec{L}_z = I\vec{\omega}$ względem osi z

- pyłki słonek z trampoliny } ale energia kinetyczna nie jest stała
- łyżwiarka

bo $E_k = \frac{1}{2}I\omega^2 > \frac{1}{2}I_0\omega_0^2 = E_0$ bo $I\omega = I_0\omega_0$

$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I_0\omega_0^2} = \frac{(I\omega)^2}{(I_0\omega_0)^2}\frac{I_0}{I} = \frac{I_0}{I}$ $I_0 \approx I$!

- **precesja** osi Ziemi (okres po 26 tys. lat) - wpływ Księżyca i Słońca
- **nutacja** (drgania osi obrotu przy precesji)!

STATYKA!